

Optimal harvesting of stochastic spatial resources

Christopher J. Costello* and Stephen Polasky†

June 23, 2006

Abstract

We extend the theory of optimal harvesting of a renewable resource to a generalized stochastic setting with spatial externalities. Doing so makes it possible to address a range of important questions for which existing theory is inadequate. Results indicate that assigning private property rights to resource “patches” is insufficient to achieve economic efficiency. The first-best spatial harvest pattern typically includes some areas completely closed to harvest; this result is robust to (and in fact does not even require) different sources of stochasticity.

Analysis of the spatial distribution of economic activity has increased significantly in recent years. Prominent applications include the spatial dimensions of international trade and regional development (Fujita et al. 2001), locational equilibrium in urban growth (Epple and Sieg 1999), and natural resource extraction (Kolstad 1994; Sanchirico and Wilen 1999; Gaudet et al. 2001). These applications have emerged from the realization that resources and economic opportunities are distributed heterogeneously across space, giving rise to issues of transportation, locational choice, and trade. In addition to exhibiting spatial heterogeneity, many biological resources move across space, thus connecting actions in one place to future economic opportunities in other places. As such, many renewable resources are both spatially heterogeneous and linked across space. While these characteristics have been recognized (e.g. (Sanchirico and Wilen 2005)) their consequences have not been fully internalized or explored in economic analyses.

To do so, we extend the theory of resource extraction to the stochastic spatial domain, capturing both spatial heterogeneity and spatial connections. Spatial heterogeneity can arise from economic forces (e.g. harvest or transportation cost) or resource constraints (e.g. differential productivity); this facet has been partially examined by Gaudet et al. (2001). While ubiquitous in the real world, analysis of spatial externalities (e.g. spatial stock depletion in oil and water extraction or mobile

*Corresponding Author: Department of Economics and Bren School, 4410 Bren Hall, Santa Barbara, CA 93117, Costello@bren.ucsb.edu

†Department of Applied Economics, University of Minnesota

fish stocks) has been limited to case studies and steady state analysis, both of which significantly limit the scope and generality of economic insight.¹

Our model generalizes renewable resource economics to the spatial domain with an arbitrary number and location of heterogeneous resource production sites (called “patches”). Biological production is both spatially heterogeneous and stochastic, as is dispersal (i.e. resource connectivity) across space. Economic variables are spatially heterogeneous. Solving this model involves stochastic spatial dynamic optimization with arbitrary spatial heterogeneity and arbitrary spatial externalities. Despite model complexity we obtain sharp analytical results and show how existing economic theories fall out as special cases.

Within this framework we characterize the optimal spatially-explicit harvest strategy that maximizes the present value of profit from harvest. We show how the “golden rule of growth” extends to optimal harvesting with multiple patches. We derive the fully-coordinated spatial optimum which, in general, differs from the solution that would be achieved by uncoordinated private property owners.

This new theory also allows us to examine an important policy question regarding spatial resource use. Could closing some areas to harvest (i.e. by establishing biological “reserves”) actually increase profits? While this does not immediately accord with economic intuition, we find in the affirmative, and that this result hinges on the strength of the spatial heterogeneity and externalities. Provided that either of these effects is sufficiently strong, reserves will always be a part of an economically optimal solution. Importantly, reserves must be optimally sited to achieve these economic benefits; suboptimally sited reserves will reduce profits. We demonstrate that the optimal harvesting strategy in non-closed areas, given that some patches are optimally closed, is to decrease harvest relative to the case where no areas are closed. On the other hand, if some areas are arbitrarily closed, then the optimal harvest strategy in non-closed areas is to increase harvest. We show that significant environmental stochasticity is sufficient, but not necessary, for spatial harvest closures to achieve a first best economic solution.

Our focal resource is the fishery which is well characterized by spatial connectivity (larval dispersal across space) and heterogeneity (sites of differing harvest costs or biological productivity). Fisheries are also subject to significant interannual variability - both in life history stages and in the dispersal process itself. In addition to mapping to fisheries problems, the theory developed here is applicable to other renewable resources (e.g., forest products) as well as important policy issues that share many formal similarities with renewable resources (e.g., antibiotic or pesticide resistance).

The paper is organized as follows. We begin with a discussion of prior bioeconomics literature and institutions governing renewable resource use (Section 1). The

¹Recent exceptions in the ecology literature include Sanchirico et al. (2006) and Grafton et al. (2005), both of which divide the ocean into two sites and examine the bioeconomic consequences of closing one of those sites to harvest. Those studies do so for particular functional forms governing biological growth and economic payoffs, and use simulations to illustrate results. The focus of the former is on exploring the conditions under which closing one patch is optimal. The focus of the latter is to show that reserves can make a system more resilient following a discrete negative shock to the ecosystem.

main economic insights can be gleaned from a simple example (Section 2). We provide a general model in Section 3 and derive our major results in Section 4. Section 5 provides a brief discussion of issues including the market institutions that may give rise to optimal reserve siting.

1 Background

Fifty years ago, scientists were beginning to recognize that many renewable resources, once plentiful and seemingly limitless, were in decline; stocks were diminishing and increasing amounts of effort were required to maintain harvest levels. At the time, biologists played the leading role in policy design and analysis; primarily focused on fisheries. Only later would economists engage in this discussion and convincingly articulate the role economic behavior played in the problem, and the potential role economic institutions could play in the solution (Gordon 1954; Scott 1955). As H.S. Gordon explained:

Owing to the lack of theoretical economic research, biologists have been forced to extend the scope of their own thought into the economic sphere and in some cases have penetrated quite deeply, despite the lack of the analytical tools of economic theory. (Gordon 1954).

The seminal works of Gordon (1954) and Scott (1955) spawned an immense economics literature more or less devoted to examining the institutional failures inherent in competitive resource extraction. Gordon (1954) illuminated the externality of one harvester on others, while Scott (1955) was the first to note the dynamic nature of the problem; current harvest had an effect on future stocks. When combined with a reasonable depiction of economic harvesting behavior, these observations pointed out the “tragedy of open access”. In the absence of certain kinds of institutions, rents would be completely dissipated and the value of the fishery driven to zero. Subsequent works by Crutchfield and Zellner (1962), Smith (1968), Smith (1969), Clark and Munro (1975), and others examined this dynamic interplay in detail, and outlined a number of possible institutional corrections, which, it was thought, could help secure rents in perpetuity. The subsequent literature on bioeconomics examined a number of extensions to the basic model including rational expectations (Berck and Perloff 1984), environmental variability (Reed 1979), overcapitalization (Grafton et al. 1996), political economy (Johnson and Libecap 1982), and others.²

Five decades hence, despite countless subsequent contributions by economists, many renewable resources are – by any performance measure – patently worse-off than they were in the 1950’s (Myers and Worm 2003; Jackson et al. 2001). And just as Gordon observed in 1954, biologists are playing policy analysts, and are, in fact, leading inquiry about the linkages between scientific insights and the design of institutions for managing these systems. As before, most of the analysis by biologists on this issue takes little account of economic behavior, incentives, and objectives.

²Wilén (2000) provides an informative and thorough chronology of the contributions of economists to institutional policy design for natural resources.

Spatial connectivity of the bioeconomic environment – driven by the interplay between environmental, biological, and economic conditions – imposes an important spatial externality that remains largely ignored in economic analysis *but is perhaps as significant a cause of misallocation of resources as the dynamic externality identified five decades ago*. Spatial externalities arise whenever economic activity in one location influences returns in another location. If fish larvae drift, seeds disperse, water tables recede, or pests intermingle, then optimal spatial extraction may differ from that which arises from the decentralized private property solution.

Would accounting for these complex dynamical and often stochastic spatial linkages appreciably change, in a qualitative way, the conclusions about optimal economic exploitation of natural resources? That is the focus of this paper.

To our knowledge the first substantive attempt to link spatial relationships in a true bioeconomic model is in Clark (1990), which explores both open access and optimized harvest in a model where spatial connections are driven by diffusion. Brown and Roughgarden (1997) were the first to examine a metapopulation model in an economic optimization framework, and instead of smooth diffusion, they assume uniform connectivity across space. Assuming diffusion along a line or a common larval pool trivializes the spatial linkages and thus significantly limits the scope of economic questions that can be addressed. We generalize the spatial connections by allowing a patchy environment in which spatial connections are more realistic. Treating spatial externalities explicitly allows us to derive qualitatively different conclusions.

A series of papers by James Sanchirico and James Wilen on the bioeconomics of marine reserves in fisheries represents the most comprehensive investigation of the bioeconomics of spatial fisheries to date. Sanchirico and Wilen (1999) outline a metapopulation model similar to Brown and Roughgarden (1997) in a discrete patchy environment. The focus of Sanchirico and Wilen (2001) was to examine the consequences of establishing a reserve in the absence of any regulation in the harvest region. Open access outside the reserve drives rents to zero, so the authors examine the consequences of reserve creation on total harvest.

But given our interest in *optimal* spatial exploitation, this small economics literature that focuses on open access conditions provides little guidance; though some progress has been made. Using a mix of 2-patch examples, specific functional forms, and simulation, fisheries economists have partially analyzed the economic consequences of marine reserves on fisheries profits. Conrad (1999) and Hannesson (1998) reach pessimistic conclusions about the ability of reserves to increase profitability while Sanchirico et al. (2006) find that reserves can increase profits.

Smith and Wilen (2003) examine the economic implications of closing a patch to fishing, paying particular attention to fishermen's decisions about whether and where to fish. They find that taking these spatial decisions into account can significantly diminish the attractiveness of area closures. We are aware of only one paper that examines optimal spatial exploitation in a generalized connected and patchy environment. Sanchirico and Wilen (2005) analyze the question by examining the case of "regulated open access" in which the fishery manager can choose spatially heterogeneous landings and effort taxes in a deterministic environment. In that model the objective is linear in these control variables and so a bang-bang solution is obtained.

Focus is devoted to the singular control that obtains in the equilibrium. The scope of that paper is limited to interior solutions which leaves unanswered the question of whether harvest closures can ever be a part of a spatial optimized harvest regime.

Our paper generalizes and contributes to the existing literature along 3 important dimensions. First, we will analyze in a general model, the optimal spatial harvest, accounting for the important case of some patch closures. Second, we solve for the optimal harvest dynamics that account for spatial externalities. Finally, while we generalize all results to a stochastic setting, we will show that a highly variable environment is sufficient, though not necessary, for biological reserves to emerge as part of the optimal solution to the spatial harvest problem. We begin, in the next section, with an intuition-building example.

2 A simple example

Much of the intuition for our main results can be gleaned from a simple two-patch example. Suppose a single fisherman, whose goal is to maximize the present value of profit from fishing, has control over a closed system consisting of two patches, A and B . For this example, assume harvest cost is linear in harvest and price is constant so that profit is linear in harvest. With these assumptions, the optimal harvest plan, which maximizes the present value of profit, is one that maximizes the present value of harvest volume. Harvesting takes place in discrete periods and let δ be the discount factor between periods: $\delta = 1/(1+r)$, where r is the discount rate. Define x_{it} as the fish stock in patch i at the beginning of period t , and h_{it} as the harvest in patch i , in period t , $i = A, B$, $t = 0, 1, 2, \dots$. The fish stock in patch i at the end of period t after harvest (called “escapement”) is: $e_{it} = x_{it} - h_{it}$. Between periods, the fish stock grows. The growth function in each patch is given by a continuous increasing and concave function of escapement: $f(e_{it})$.³ Because of ocean currents, fish migrate from patch A to patch B . Assume that all fish in patch B at the end of period t start period $t+1$ in patch B , and that some fraction θ of fish in patch A at the end of period t migrate to patch B at the start of period $t+1$. Given this, the equations of motion for stocks in the two patches are:

$$x_{At+1} = (1 - \theta)f(e_{At}) \tag{1}$$

$$x_{Bt+1} = \theta f(e_{At}) + f(e_{Bt}) \tag{2}$$

As a benchmark, consider the case in which these patches are completely independent (i.e. $\theta = 0$). In this case, we can apply standard economic intuition to derive optimal escapement in each patch independently: In each patch the optimal escapement, e^* , is characterized by the stock level at which the rate of growth of the fish stock (the biological rate of return) equals the financial rate of return:

$$f'(e^*) = 1/\delta \tag{3}$$

³We assume that $f'(0) > 1/\delta$. If not, it would be optimal to fish to extinction and simply invest returns in a financial asset (Clark 1973).

This result is the standard “golden rule” of growth as applied to resource economics. This result holds whenever there is positive harvest. If the fishery begins a period with depleted stock such that $x_{it} < e^*$, then it is optimal to close the fishery that period because the biological return from leaving fish in the ocean is greater than the financial rate of return. Such closures, however, would only be temporary, allowing depleted stocks to replenish. In steady state, optimal harvest would be positive in each patch. Establishing a biological reserve that would result in permanent closure of a patch would only reduce profits.

When spatial connections exist ($\theta > 0$) it turns out that establishing a permanent biological reserve may indeed be economically optimal. The optimal strategy in each patch is still to harvest to the point where the growth rate of the resource equals the financial rate of return (as described in Equation 3). However, because fish migrate from patch A to patch B , the fish stock in patch A may be small, even when there is no harvest in patch A . Let the steady state stock in patch A in the absence of harvest be given by x_k , which is defined by: $x_k = (1 - \theta)f(x_k)$. As θ increases, x_k , decreases (as shown in Figure 1). For sufficiently large θ , $x_k < e^*$, and it will be optimal to permanently close patch A to harvesting. This is illustrated in Figure 1 for the case of high spillover, θ_1 , which implies a low steady state stock: $x_k^1 < e^*$. In this case, patch A is a biological source that should be protected. It is optimal to close the fishery in patch A because the biological return from leaving fish in patch A is greater than the financial rate of return. Some of the fish from patch A then migrate to patch B where harvest occurs. For a low spillover rate, θ_0 , $x_k^0 > e^*$, and it is not optimal to establish a biological reserve.

This simple example has shown that establishing permanent biological reserves can be optimal in the case where there are spatial connections between patches. Without spatial connections, it is never optimal to establish a permanent biological reserve. This simple example highlights the importance of spatial connections in determining the optimal harvesting regime. The simple example has made several oversimplifying assumptions. In what follows we will relax these assumptions to explore the generality of this, and other, results.

3 The spatial harvest problem

We develop a general spatial harvesting model where harvest can take place in $i = 1, 2, \dots, I$ discrete non-overlapping patches over $t = 0, 1, 2, \dots, T$ discrete time periods. Patches may be heterogeneous along economic and/or biological dimensions. The model includes stochasticity in several key biological relationships, which is an important feature of most renewable resources. In what follows, we denote random variables by capital letters and realizations of random variables by lower-case letters.

3.1 Spatial biology

The stock in patch i at the start of period t , x_{it} , is assumed to be known at time t , and initial stock in each patch, x_{i1} , is given. Harvest in patch i in period t is

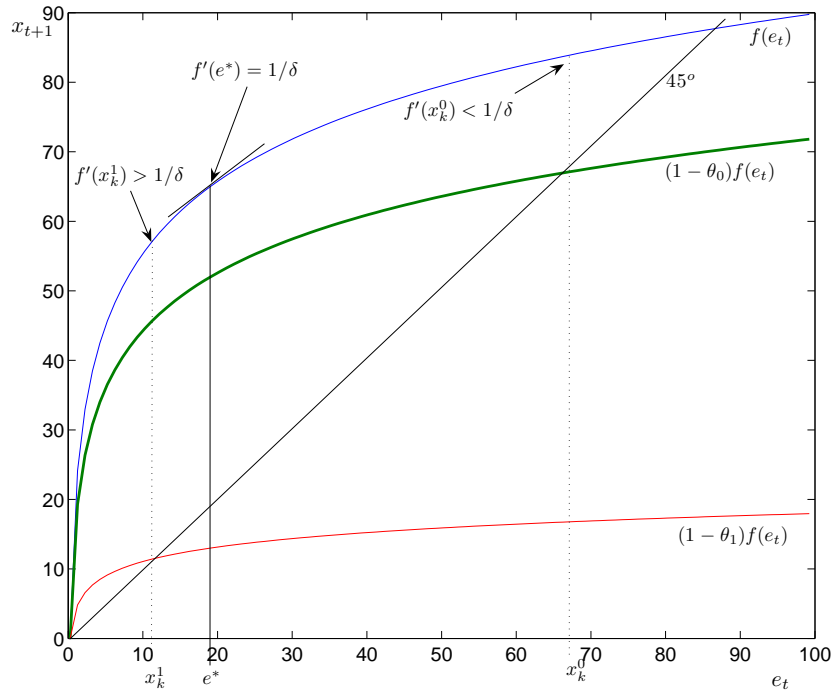


Figure 1: Biological growth in patch A is given by $f(e)$. Because fish migrate from patch A to patch B , only $(1 - \theta)f(e)$ fish remain in patch A . With no harvest, the steady state stock of fish in patch A is determined by the intersection of $(1 - \theta)f(e)$ with the 45° line. For high values of θ , the no harvest steady state stock will like to the left of e^* , as shown with high migration value θ_1 . Because $x_k^1 < e^*$, the biological return in patch A ($f'(x_k^1)$) exceeds the financial return $(1 + \delta)$ and it is optimal to close patch A to harvest.

h_{it} and is a decision variable. As in the simple example, biological growth depends on the stock of reproducing adults after harvest (escapement), which is defined as: $e_{it} = x_{it} - h_{it}$. Growth in each patch yields the stock of young, Y_{it} , which depends on a spatially distinct average growth function $f_i(e)$, with $f'_i(e) > 0$, $f''_i(e) < 0$, and $f'_i(0) > 1/\delta$, and may be influenced by stochastic processes (e.g., nutrient availability, rainfall). The number of young produced in patch i at time t is:

$$Y_{it} = Z_{it}^f f_i(e_{it}) \quad (4)$$

where Z_{it}^f is a random variable whose distribution is known and is time independent with expected value equal to 1 and support bounded below by 0. Equation 4 is a spatial version of the stochastic fishery problem considered by Reed (1979), Costello et al. (2001), and others.

The young that are produced in each patch $i = 1, \dots, I$ then disperse across space. The pattern of dispersal may be stochastic (dependent on ocean currents, wind, etc.). Denote by D_{ji} a scaled multinomial random variable indicating the percentage of young that originate in patch j that settle in patch i (so $\sum_i D_{ji} = 1$). Keeping track of all possible source locations, total settlement to patch i is:

$$S_{it} = \sum_{j=1}^I Y_{jt} D_{ji}. \quad (5)$$

Following settlement in a patch, we assume that individuals do not migrate out of that patch. The number of settlers in patch i that survive the time period to adulthood is $Z_{it}^S \sigma_i(S_{it})$, where $\sigma_i(S_{it})$ is the (possibly) density dependent average survival to adulthood in patch i and Z_{it}^S is a random variable. Adult survival from one period to the next is given by $Z_{it}^\mu \mu_i(e_{it})$, where $\mu_i(e_{it})$ is the (possibly) density dependent average survival as a function of the number of adults after harvest and Z_{it}^μ is a random variable. We assume that the distribution of Z_{it}^S and Z_{it}^μ are known, time independent, each with expected value equal to 1 and support bounded below by 0. We also assume that the random variables ($Z_{it}^f, Z_{it}^S, Z_{it}^\mu, D_{ji}$) are independent of each other.

Pulling together the various parts of the biological model, we can summarize the equation of motion describing the stock of adults in patch i in time period $t + 1$ as a random variable given by:

$$X_{it+1} = Z_{it}^\mu \mu_i(e_{it}) + Z_{it}^S \sigma_i(S_{it}) \quad (6)$$

$$= Z_{it}^\mu \mu_i(e_{it}) + Z_{it}^S \sigma_i \left(\sum_{j=1}^I Z_{jt}^f f_j(e_{jt}) D_{ji} \right) \quad (7)$$

The first term on the right hand side of Equation 6 is the stock of surviving adults from the previous period. The second term on the right hand side is the stock of new adults, which depends on reproduction and dispersal from all patches. Therefore, the stock in patch i in time period $t + 1$ may depend on escapement in all patches, e_{jt} , $j = 1, \dots, I$, and on the random variables in all patches, Z_{jt}^f and D_{ji} , $j = 1, \dots, I$, as well as patch specific random variables, Z_{it}^μ , and Z_{it}^S . The timing of growth and harvest are summarized in Figure 2.

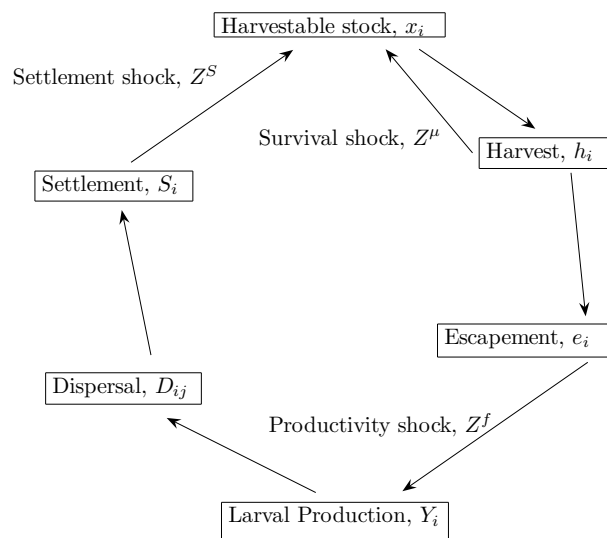


Figure 2: Timing of growth, dispersal, and harvest in a stochastic spatially connected renewable resource model.

3.2 Spatial economics

We assume an elastic demand at price p per unit harvest, and a marginal cost of harvest function, $c_i(s)$, which is a non-increasing function of the current stock, $c'_i(s) \leq 0$. By indexing $c_i(\cdot)$ by i we allow for the possibility that harvest costs may be location specific. For example marginal harvest costs in fishing may increase with depth or distance to port. The patch- i period- t payoff from harvest h_{it} starting with a population of x_{it} and ending with a population of e_{it} is: $ph_{it} - \int_{e_{it}}^{x_{it}} c(s)ds$.

The economic objective is to maximize the expected net present value of harvest from I -patches over the T -period horizon:

$$\max_{\{h_{it}\}} E \sum_{t=1}^T \delta^t \sum_{i=1}^I \left[ph_{it} - \int_{e_{it}}^{x_{it}} c(s)ds \right] \quad (8)$$

where E is the expectation operator. The maximization problem is subject to the equation of motion for stock in each patch $i = 1, 2, \dots, I$ (Equation 6), and given initial stocks x_{i1} , for all i . The objective is to identify a feedback control rule $\mathbf{h}_t^*(\mathbf{x}_t)$ that is an I -vector function of state-dependent controls that yields the optimal patch-specific harvest as a function of the vector of patch-specific stocks in any given period, t .

4 Results

In this section we derive an analytic solution to the stochastic spatial optimal harvesting problem 8. We begin by deriving an interior solution and the conditions required for its existence. We then analyze the bioeconomically relevant and interesting corner solution case where it is optimal to close at least one patch to harvesting, either temporarily or permanently. A question central to this analysis is if, and under what biological or economic conditions, such closures are optimal. Our approach - of solving for the optimal spatial harvest by the sole owner and observing whether closures emerge as a solution - allows our model to inform the optimal spatial and temporal design of such closures, taking into account the spatial externalities resulting from biological connectivity and economic heterogeneity across space.

We represent the spatial harvesting problem under uncertainty as a stochastic dynamic programming problem with \mathbf{x}_t as the period- t state vector of stocks and \mathbf{e}_t as the period- t control vector, as follows:⁴

$$V_t(\mathbf{x}_t) = \max_{\mathbf{e}_t} \sum_{i=1}^I \left[p(x_{it} - e_{it}) - \int_{e_{it}}^{x_{it}} c(s)ds \right] + \delta E_t \{V_{t+1}(\mathbf{X}_{t+1})\}. \quad (9)$$

The value function $V_t(\mathbf{x}_t)$ is unknown; in what follows we derive its salient properties. Equation 9 is subject to state transitions given by Equation 6 and initial stocks \mathbf{x}_1 . From the perspective of period- t , period $t+1$ stocks are random variables. Solving for the optimal solution requires setting the marginal value of harvest in each patch equal

⁴Because $\mathbf{e}_t = \mathbf{x}_t - \mathbf{h}_t$, and \mathbf{x}_t is known at time t , we can use either \mathbf{h}_t or \mathbf{e}_t as the control vector. It turns out to simplify the analytics to use \mathbf{e}_t .

to the discounted expected marginal value of additional escapement from the patch, where the marginal value of additional escapement is determined by its contribution to future harvests in connected patches.

In general, stochastic dynamic programming problems such as that in Equation 9 are difficult problems to solve analytically.⁵ However, we can make progress in the analysis by taking advantage of the special structure of the problem. Define \underline{x}_i as the stock level to which a myopic harvester would extract the resource: $\underline{x}_i = \max(0, \hat{x}_i)$, where \hat{x}_i is the level of stock at which marginal profit is zero, defined by $p = c(\hat{x}_i)$. Starting with a stock of x , the patch- i period- t profit from harvesting down to \underline{x}_i is given by:

$$Q_i(x) \equiv p(x - \underline{x}_i) - \int_{\underline{x}_i}^x c_i(s) ds \quad (10)$$

where s is just a dummy of integration. Using this function, we can re-write the dynamic programming Equation 9 as follows:

$$V_t(\mathbf{x}_t) = \max_{\mathbf{e}_t} \sum_{i=1}^I [Q_i(x_{it}) - Q_i(e_{it})] + \delta E_t \{V_{t+1}(\mathbf{X}_{t+1})\} \quad (11)$$

which is subject to biological state transitions given in Equation 6 and initial stocks \mathbf{x}_1 . We represent optimal solutions to this problem by $\mathbf{e}_t^*(\mathbf{x}_t)$. We assume concavity of returns in \mathbf{e}_t so that there is a unique solution $\mathbf{e}_t^*(\mathbf{x}_t)$. Under the assumptions of our model, we can guarantee concavity when $f_i''(\cdot)$ is large in absolute value (highly concave growth function) relative to $c_i''(\cdot)$.

4.1 An interior solution to the stochastic spatial harvesting problem

Making use of the form of the stochastic dynamic program in Equation 11, we can show that the optimal harvesting problem is *state separable*.

Definition 1. *An optimization problem is **state separable** if the first order conditions are independent of the state vector.*

Lemma 1. *Provided that an interior solution exists, the period t dynamic program given in Equation 11 is state separable.*

Proof. The dynamic programming problem is:

$$V_t(\mathbf{x}_t) = \max_{\mathbf{e}_t} \underbrace{\sum_{i=1}^I [Q_i(x_{it}) - Q_i(e_{it})]}_{\text{Current Payoff}} + \underbrace{\delta E_t \{V_{t+1}(\mathbf{X}_{t+1})\}}_{\text{Future Payoff}}. \quad (12)$$

⁵In fact, even solving this problem numerically would quickly become difficult if the number of patches exceeded 5 or so.

The necessary condition for an interior solution is:

$$-Q'_i(e_{it}) + \delta E_t \left\{ \sum_{j=1}^I \frac{\partial V_{t+1}(X_{t+1})}{\partial X_{jt+1}} \frac{\partial X_{jt+1}}{\partial e_{it}} \right\} = 0 \quad \forall i \quad (13)$$

The necessary condition is also sufficient given the assumption of concavity of returns in the vector of controls (escapement). To show state separability, and thus prove Lemma 1, we must show that all terms in this expression are independent of x_{it} . The first term, which reflects the marginal contribution of escapement to current period payoff, is independent of x_{it} by inspection. The term inside the expectation operator is the product of the partial derivative of the value function with respect to X_{jt+1} and the partial derivative of X_{jt+1} with respect to e_{it} . Note that in an interior solution $e_{it} < x_{it}$. Using the definition of the evolution of stock (Equation 6) we note that X_{it+1} is a function of e_{it} but not of x_{it} . Therefore, the terms in the bracket are independent of x_{it} . Since all terms in Equation 13 are independent of x_{it} , the period t problem is state separable. \square

The key insight here is that for a *state separable* problem the current period choice of escapement does not depend on current stock. Likewise, next period's escapement choice will not depend on the stock. Therefore, optimal escapement is determined by a simple comparison between the current returns and the expected returns from increasing stocks in the next period, but does not depend on expected future returns in periods farther into the future. With interior solutions, the dynamic programming problem can be broken into a series of non-trivial one-period problems. This feature renders our problem tractable and allows us to characterize an optimal solution to the stochastic spatially explicit bioeconomic problem.

An important economic insight that follows from the state separability property is captured in the following proposition.

Proposition 1. *If an interior solution to the dynamic programming equation exists, the optimal feedback control rule will be both time and state independent and will, in general, vary across space.*

Proof. The necessary condition for an interior optimal solution to the dynamic programming equation (Equation 11) for patch i at time t is given by Equation 13. Note that e_{it}^* is independent of x_{it} by Lemma 1. Therefore, a change in stock in the next period affects the value function in $t+1$ only through terms $Q_j(x_{jt+1})$, for $j = 1, \dots, I$. Using this fact along with the state transition equations (Equation 6), we can rewrite the necessary condition for patch i at time t as follows:

$$-Q'_i(e_{it}^*) + \delta E_t \left\{ Q'_i(x_{it+1}) Z_{it}^\mu \mu'_i(e_{it}^*) + \sum_{j=1}^I Q'_j(x_{jt+1}) Z_{jt}^S \sigma'_j(\cdot) Z_{it}^f f'_i(e_{it}^*) D_{ij} \right\} = 0 \quad (14)$$

Since the distribution of shocks is independent of time, as is biological growth, dispersal and economic returns, the optimal choice, e_{it}^* , is independent of time. However,

since biological growth, dispersal and economic returns can vary across patches, the optimal choice can, in general, vary across space. \square

Proposition 1 characterizes the optimal interior solution to the spatial harvesting problem under uncertainty. The optimal interior solution is to identify a patch-specific escapement level, and to harvest down to that level every period. For a given patch, that optimal escapement level remains fixed over time. Environmental fluctuations that affect initial stock size in a patch will affect harvest but not optimal escapement.⁶ This is a spatial generalization of a result derived by Reed (1979). Inspecting the proof to Proposition 1, these optimal escapement levels can vary across space for three reasons. First, spatial heterogeneity in the economic environment (captured here by different harvest costs) can drive spatial differentiation of harvest. Second, spatial heterogeneity in the biological environment (captured by differences in biological productivity across patches) will influence harvesting. Finally, and perhaps most importantly, patterns of dispersal, which connect the biological functions of different patches, can affect harvest. These spatial connections are what distinguish this problem from similar analyses in aspatial environments and can play an important role in determining the optimal harvest strategy. However, there is a set of special conditions under which spatial connectivity plays no role in the interior solution.

Condition 1. *Total harvest cost is a linear function of total harvest (given by ch_{it} in patch i , period t).*

Condition 2. *The survival function $\sigma_j(x)$ is linear and identical across patches (so $\sigma_j(x) = \sigma x$.)*

Proposition 2. *Under Conditions 1 and 2, and provided that an interior solution to the dynamic programming equation exists, the optimal feedback control rule satisfies the golden rule of growth in each patch in each time period and is independent of dispersal.*

Proof. Under Condition 1, $Q_i(x) = (p - c)(x - \underline{x}_i)$. Under Condition 2, the necessary condition for an interior solution to the optimal feedback rule for patch i at time t is:

$$-(p - c) + \delta E_t \left\{ (p - c) Z_{it}^\mu \mu'_i(e_{it}^*) + \sum_{j=1}^I (p - c) Z_{jt}^S \sigma Z_{it}^f f'_i(e_{it}^*) D_{ij} \right\} = 0 \quad (15)$$

Simplifying, we obtain: $1 = \delta \left\{ \mu'_i(e_{it}^*) + \sum_{j=1}^I \sigma f'_i(e_{it}^*) D_{ij} \right\}$. And since $\sum_{j=1}^I D_{ij} = 1$, this expression simplifies to:

⁶Note the distinction here between a policy of constant *escapement* and a policy of constant *harvest*. We have shown that the optimal policy in a stochastic environment is to achieve a constant escapement level. This means that the optimal amount of harvest in any given patch will vary from year to year, and will be state-dependent.

$$1/\delta = \mu'_i(e_{it}^*) + \sigma f'_i(e_{it}^*). \quad (16)$$

Optimal escapement in a patch, as characterized by Equation 16, is independent of dispersal. \square

The right hand side of Equation 16 is the discounted expected growth of the stock remaining at the end of the period. Equation 16 can be interpreted as a golden rule of growth. Dispersal drops out and does not affect the optimal rule. With identical costs and survival across sites, what matters is the productivity of each site ($\mu'_i(e_{it}^*) + \sigma f'_i(e_{it}^*)$), not where recruits end up. No matter where recruits end up, they are harvested and generate the same returns per unit, $(p - c)$.

In general, with either non-linear density dependent survival ($\sigma_i(S_{it})$), differences in survival rates across patches, or non-linear harvest costs, dispersal will play a role in optimal escapements. We focus on the more general case (i.e. without assuming conditions 1 and 2) in what follows.

4.2 Corner solutions: a case for biological reserves?

Over the past several decades there has been a major expansion of protected areas (“reserves”) in which extractive economic activities, such as timber harvesting, hunting or fishing, are banned or restricted. Currently about 10% of the earth’s land area (almost twice the size of Europe or Australia)⁷ and 5% of the territorial oceans (about 20 times the area of the Great Lakes)⁸ are in reserves (authors’ calculations based on data in World Database on Protected Areas (2005)).

One justification for expanding reserves is to achieve biological objectives; reserves are a means to conserve biodiversity. However, a stronger claim is often made that reserves increase the value of extractive economic activity. These claims are largely unsubstantiated by economic analysis; the few exceptions noted earlier in this paper. For example, in justifying marine reserves, biologists have claimed that not only is the reserve good for the fish, but it is good for the fisherman (Hastings and Botsford 1999). That this is so does not immediately accord with economic intuition. Provided that the initial stock size in every patch in every period is sufficiently large, an interior solution (in which there is positive harvest in every patch in every period) is optimal. But with stochasticity and spatially connected patches there is no guarantee that initial stock size in every patch in every period will be sufficient large. In this section we focus on corner solution cases where it is optimal to close a patch, either temporarily or permanently, to harvest. We begin with the following result.

Proposition 3. *Patch i should be closed to harvesting in period t if and only if $x_{it} < \bar{e}_{it}$, where \bar{e}_{it} satisfies the following implicit equation:*

$$-Q'_i(\bar{e}_{it}) + \delta E_t \left\{ \sum_{j=1}^I \frac{\partial V_{t+1}(X_{t+1})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} \right\} = 0 \quad (17)$$

⁷Total land area is 1.48E10 ha, and total terrestrial reserves constitute 1.47E9 ha.

⁸Total territorial and EEZ ocean area is 1.07E10 ha, and total marine protected area is 4.8E8 ha.

Proof. Because $-Q'_i(e) < 0$, and

$$\frac{\partial}{\partial e_{it}} \left\{ E_t \left(\sum_{j=1}^I \frac{\partial V_{t+1}(X_{t+1})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} \right) \right\} < 0$$

we have

$$-Q'_i(e_{it}) + \delta E_t \left\{ \sum_{j=1}^I \frac{\partial V_{t+1}(X_{t+1})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} \right\} > 0 \quad (18)$$

for $e_{it} < \bar{e}_{it}$. In this case, it is optimal to increase escapement. However, we know that $e_{it} \leq x_{it}$, so if $x_{it} < \bar{e}_{it}$, the maximum e_{it} that can be attained is $e_{it} = x_{it}$, which occurs with zero harvest. Therefore, for $x_{it} < \bar{e}_{it}$ it is optimal to close patch i to harvesting in period t . For $e_{it} > \bar{e}_{it}$, $-Q'_i(e_{it}) + \delta E_t \left\{ \sum_{j=1}^I \frac{\partial V_{t+1}(X_{t+1})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} \right\} < 0$ and it is optimal to lower escapement (increase harvest). When $x_{it} \geq \bar{e}_{it}$, it is optimal to have positive harvest and have escapement of $e_{it} = \bar{e}_{it}$. \square

The escapement target \bar{e}_{it} is the level at which marginal benefit from harvest just equals the spatially-aggregated discounted expected marginal benefit from additional escapement. If the stock, x_{it} falls below \bar{e}_{it} , then the patch should be closed in that period. It follows that if $x_{it} < \bar{e}_{it}$ for all t then patch i should be permanently closed to harvest. In that case, patch i would be a permanent biological reserve. The example in section 2 with large value of θ illustrates this possibility. With stochastic shocks to recruitment or the adult population, it is possible that $x_{it} < \bar{e}_{it}$ in some periods but not permanently, in which case it is optimal to have a temporary harvest ban, i.e. a temporary reserve, in a particular set of patches to allow stock recovery, but not institute a permanent biological reserve.

A result that follows immediately from this discussion is that patches where stocks have been depleted by past harvesting (such that $x_{it} < \bar{e}_{it}$) should be closed to harvesting, at least temporarily until stocks have recovered to a point where this inequality no longer holds.

Proposition 3 provides a necessary and sufficient condition for a harvest closure to be economically optimal. Under such a circumstance the fishery owner is still faced with the task of determining optimal harvest outside the closed area. This question is of central importance to policy surrounding protected areas and their design, yet it has received only scant attention in the literature. When analyzing the consequences of harvest closures two different approaches have been taken. Biologists typically assume maximal harvest outside the reserve (Hastings and Botsford 1999). Of course in a world with stock-dependent harvest costs (such as assumed in this model) it would never be economically rational to harvest to extirpation in a patch (provided $c(0) > p$). Another approach is to assume open access outside the reserve (e.g. Sanchirico and Wilen (2001)). In either case, reserves may be better than no reserves because their implementation partially addresses the failures associated with over-harvesting. But given our interest in optimal spatial harvesting, the following propositions analyze how implementation of a reserve will affect *optimal* harvest outside the reserve. To assist in model tractability, we will adopt the following condition for this analysis.

Condition 3. *The only patch that is possibly closed to harvest at time $t + 1$ is patch k .*

Proposition 4. *Under Condition 3, if there is some non-zero probability that it will be optimal to close patch k to harvest at time $t + 1$, then for any patch i for which $\Pr(D_{ik} = 0) < 1$, optimal escapement in patch i at time t will be higher than an equivalent case with zero probability of optimal closure of patch k at time $t + 1$.*

Proof. When patch k is in an interior solution at time $t + 1$, the marginal value of escapement from patch i in period t is:

$$\delta E_t \left\{ Q'_i(x_{it+1}) Z_{it}^\mu \mu'_i(e_{it}^*) + \sum_{j=1}^I Q'_j(x_{jt+1}) Z_{jt}^S \sigma'_j(\cdot) Z_{it}^f f'_i(e_{it}^*) D_{ij} \right\} \quad (19)$$

When patch k is in a corner solution at time $t + 1$, the marginal value of escapement from patch i in period t is:

$$\begin{aligned} & \delta E_t [Q'_i(x_{it+1}) Z_{it}^\mu \mu'_i(e_{it}^*) + \sum_{j \neq k} Q'_j(x_{jt+1}) Z_{jt}^S \sigma'_j(\cdot) Z_{it}^f f'_i(e_{it}^*) D_{ij} + \\ & \delta E_{t+1} \left[\sum_{j=1}^I Q'_j(x_{jt+2}) Z_{jt}^S \sigma'_j(\cdot) Z_{kt}^f f'_k(x_{kt+1}) Z_{kt}^S \sigma'_k(\cdot) D_{jk} Z_{it}^f f'_i(e_{it}^*) D_{ik} \right]] \end{aligned} \quad (20)$$

When patch k is in a corner solution in period $t + 1$,

$$Q'_k(x_{kt+1}) < \delta E_{t+1} \left[\sum_{j=1}^I Q'_j(x_{jt+2}) Z_{jt}^S \sigma'_j(\cdot) Z_{kt}^f f'_k(x_{kt+1}) D_{jk} \right] \quad (21)$$

Using this fact and comparing Equations 19 and 20, it follows that the marginal value of escapement from patch i in period t is higher for the case when patch k is in a corner solution in period $t + 1$ than when patch k is in an interior solution. Therefore, to satisfy the necessary conditions for an optimal solution in patch i in period t , escapement must be higher for the case where patch k is in a corner solution than when it is in an interior solution in period $t + 1$. \square

Proposition 5. *Under Condition 3, if it is optimal to have an interior solution (positive harvest) but harvest is closed in patch k at time $t + 1$, and assuming $\Pr(D_{ik} = 0) < 1$, then optimal escapement in patch i at time t will be lower than when there is no harvest closure in patch k at time $t + 1$.*

Proof. When harvest is allowed in patch k at time $t + 1$, the marginal value of escapement from patch i in period t is as shown in Equation 19. When harvest is closed in patch k in period $t + 1$, the marginal value of escapement from patch i in period t is

as shown in Equation 20. In this case the inequality is reversed as compared to the proof of Proposition 4

$$Q'_k(x_{kt+1}) > \delta E_{t+1} \left[\sum_{j=1}^I Q'_j(x_{jt+2}) Z_{jt}^S \sigma'_j(\cdot) Z_{kt}^f f'_k(x_{kt+1}) D_{jk} \right] \quad (22)$$

so that the marginal value of escapement from patch i in period t that goes to patch k in period $t+1$ is lower with the closure. Therefore, to satisfy the necessary conditions for an optimal solution in patch i in period t , escapement must be lower for the case where patch k is closed in period $t+1$ than when it is not. \square

As a consequence of Proposition 3, it is optimal to close a patch to harvest when the marginal productivity in the closed patch exceeds the financial rate of return. When other patches contribute (through dispersal) to settlement in the closed patch, this contribution has high returns. Therefore, it is optimal to allow higher escapement (lower harvest) outside the optimally designed reserve (Proposition 4). On the other hand, when a patch is closed arbitrarily (i.e., it is closed when it is not optimal to do so), the patch will have low productivity. Other patches that contribute to settlement in this patch through dispersal will have lower returns. Therefore, it is optimal to allow lower escapement (higher harvest) outside the (suboptimally designed) reserve (Proposition 5).

5 Discussion

Built on the recognition that spatial *connectivity*, *heterogeneity*, and *stochasticity* are economically important, we have extended the economic theory of renewable resource extraction to the spatial domain under uncertainty. In so doing we have uncovered a novel set of economic insights about optimal harvesting. When resource patches are connected, spatial externalities may exist that render inefficient a spatial property rights institutional design. We solve for the efficient use of resources across space, by internalizing the effects of the spatial externality. The second type of insight has to do with the effects of reserves (areas closed to harvest) on economic performance. Biologists almost unanimously favor reserves (e.g. marine protected areas) as a natural resource management tool on the largely unsubstantiated grounds that they can lead to economic gains. Yet this policy does not immediately accord with economic intuition. Might reserves deliver a first-best solution to an optimal spatial harvesting problem?

Somewhat surprisingly, we found that reserves can, in fact, yield the first-best economic outcome. This result can obtain under a number of different, and realistic, bioeconomic conditions and is robust to (even strengthened by) stochasticity within the system. Heterogeneous economic conditions (e.g. high marginal harvest cost in a region) can lead to optimal spatial closures. This result is consistent with new theory on optimal harvesting of non-renewable resources (Gaudet et al. 2001). Perhaps more interestingly, even in a spatially homogeneous economic and biological environment, certain patterns of spatial connectivity (e.g. low dispersal to a patch) can generate

sufficiently large marginal productivity to make the net marginal value of harvest in that patch negative. In such cases, the patch is optimally closed to harvest. This result can also be obtained as a result of environmental variability or shocks to the dispersal between patches, though stochasticity is not required. Patch closures can be either permanent (e.g. in the case of biological sources and sinks of larval dispersal) or temporary (e.g. in the case of bad draws from the random environment).

Maintaining our focus on harvest closures in particular patches, we also examined what harvest should obtain outside those closures. When harvest closures are optimal (as in the cases identified above), optimal harvest outside those patches is decreased to take advantage of the high marginal productivity of those patches. This is in direct opposition to the existing models of marine reserve creation which assume complete harvest outside reserves. On the other hand, if reserves are sub-optimally located (i.e. in places in which marginal productivity is low), optimal harvest outside the reserves should actually increase, relative to the case in which the patch was not closed. In general these results corroborate the largely unsubstantiated basic result from biological science that reserves may benefit fisheries. But the formal treatment of this problem outlined in this paper also provides a platform for more meaningful analysis of optimal spatial management in the presence of spatial externalities.

We have presented a relatively general spatial and dynamic economic model, have generalized it to include stochasticity, and have identified an analytical solution when an interior solution exists, and some salient characteristics of the solution when an interior solution does not exist. But this analytical tractability requires a number of limiting assumptions. An important technical requirement for us to identify a solution analytically is that the resource owner can measure without error the harvestable stock prior to harvest. If the stock was unknown, the problem would become less tractable. However, this problem could, in principle, be overcome by exploiting the fact that the stock dependent harvest costs can reveal information about the stock size. In such cases, then, a spatially heterogeneous tax could be used to achieve the optimal escapement levels derived herein (Weitzman 2002). Another somewhat limiting assumption is that the marginal economic conditions are independent of the amount harvested. This assumption is standard in the literature and buys us the *state separability* property, but may not be realistic in all natural resource contexts. In those cases, optimal harvest will have to be smoothed over time, leading to state-dependent escapement levels. Incorporating these extension is an important next step in the assessment of optimal spatial harvest of renewable resources.

While our results shed new light on an old and increasingly important problem, they also raise new questions about the institutions required to implement them. We have presented the analysis from the perspective of the sole owner with perfect and costless tenure of the entire spatial extent of the resource. More realistically spatial property rights could be assigned (say patch-by-patch) to users of the resource. Then the set of conclusions drawn here could help guide coordination between those users to achieve a first-best outcome (e.g. to implement a closure in one owner's patch). Without such coordination, we cannot expect the emergence of efficient spatial resource extraction (Hansen and Libecap 2004).

Our focus on optimal harvesting with spatial externalities facilitates analysis of a

current policy question regarding reserves and their economic implications. The rapid worldwide increase in reserve designation is driven in part by a largely unsubstantiated assumption that creating reserves can increase profit from harvest. While we have shown that reserves can indeed increase profits, our analysis also emphasizes that careful design, by incorporating *economic*, rather than just *biological* reasoning, is essential to their success and efficiency.

References

- Berck, P. and J. Perloff (1984). An open-access fishery with rational expectations. *Econometrica* 52(2), 489–506.
- Brown, G. and J. Roughgarden (1997). A metapopulation model with private property and a common pool. *Ecological Economics* 22, 65–71.
- Clark, C. (1973). Profit maximization and the extinction of animal species. *Journal of Political Economy* 81, 950–961.
- Clark, C. and G. Munro (1975). The economics of fishing and modern capital theory: a simplified approach. *Journal of Environmental Economics and Management* 2, 92–106.
- Clark, C. W. (1990). *Mathematical Bioeconomics* (2 ed.). John Wiley.
- Conrad, J. (1999). The bioeconomics of marine sanctuaries. *Journal of Bioeconomics* 1, 205–217.
- Costello, C., S. Polasky, and A. Solow (2001). Renewable resource management with environmental prediction. *Canadian Journal of Economics* 34(1), 196–211.
- Crutchfield, J. and A. Zellner (1962, April). Economic aspects of the Pacific halibut fishery. *Fishery Industrial Research* 1(1).
- Epple, D. and H. Sieg (1999). Estimating equilibrium models of local jurisdictions. *Journal of Political Economy* 107(4), 645–681.
- Fujita, M., P. Krugman, and A. Venables (2001). *The Spatial Economy: Cities, Regions, and International Trade*. MIT Press.
- Gaudet, G., M. Moreaux, and S. Salant (2001). Intertemporal depletion of resource sites by spatially distributed users. *The American Economic Review* 91(4), 1149–1159.
- Gordon, H. (1954). The economic theory of a common property resource: the fishery. *Journal of Political Economy* 62, 124–142.
- Grafton, Q., D. Squires, and J. Kirkley (1996). Private property rights and crises in world fisheries: Turning the tide? *Contemporary Economic Policy* 14(4), 90–99.
- Grafton, R. Q., T. Kompas, and D. Lindenmayer (2005). Marine reserves with ecological uncertainty. *Bulletin of Mathematical Biology* 67, 957–971.

- Hannesson, R. (1998). Marine reserves: what would they accomplish? *Marine Resource Economics* 13, 159–170.
- Hansen, Z. and G. Libecap (2004). Small farms, externalities, and the dust bowl of the 1930s. *Journal of Political Economy* 112(3), 665–694.
- Hastings, A. and L. W. Botsford (1999, May). Equivalence in yield from marine reserves and traditional fisheries management. *Science* 284, 1537–1538.
- Jackson, J., M. Kirby, W. Berger, K. Bjorndal, L. Botsford, B. Bourque, R. Bradbury, R. Cooke, J. Erlandson, J. Estes, T. Hughes, S. Kidwell, C. Lange, H. Lenihan, J. Pandolfi, C. Peterson, R. Steneck, M. Tegner, and R. Warner (2001). Historical overfishing and the recent collapse of coastal ecosystems. *Science* 293, 629–637.
- Johnson, R. and G. Libecap (1982). Contracting problems and regulation: the case of the fishery. *American Economic Review* 72(5), 1005–1022.
- Kolstad, C. K. (1994). Hotelling rents in Hotelling space: product differentiation in exhaustible resource markets. *Journal of Environmental Economics and Management* 26, 163–180.
- Myers, R. and B. Worm (2003). Rapid worldwide depletion of predatory fish communities. *Nature* 423, 280–283.
- Reed, W. (1979). Optimal escapement levels in stochastic and deterministic harvesting models. *Journal of Environmental Economics and Management* 6, 350–363.
- Sanchirico, J., U. Malvadkar, A. Hastings, and J. Wilen (2006). When are no-take zones an economically optimal fishery management strategy? *Ecological Applications In Press*.
- Sanchirico, J. and J. Wilen (1999). Bioeconomics of spatial exploitation in a patchy environment. *Journal of Environmental Economics and Management* 37, 129–150.
- Sanchirico, J. and J. Wilen (2001). A bioeconomic model of marine reserve creation. *Journal of Environmental Economics and Management* 42, 257–276.
- Sanchirico, J. and J. Wilen (2005). Optimal spatial management of renewable resources: matching policy scope to ecosystem scale. *Journal of Environmental Economics and Management In Press*.
- Scott, A. (1955). The fishery: the objectives of sole ownership. *Journal of Political Economy* 63, 116–124.
- Smith, M. and J. Wilen (2003). Economic impacts of marine reserves: the importance of spatial behavior. *Journal of Environmental Economics and Management* 46(2), 183–206.
- Smith, V. (1968). Economics of production from natural resources. *The American Economic Review* 58, 409–431.

- Smith, V. (1969). On models of commercial fishing. *Journal of Political Economy* 77, 181–198.
- Weitzman, M. (2002). Landing fees vs harvest quotas with uncertain fish stocks. *Journal of Environmental Economics and Management* 43(2), 325–338.
- Wilen, J. (2000). Renewable resource economists and policy: what differences have we made? *Journal of Environmental Economics and Management* 39, 306–327.
- World Database on Protected Areas (2005). <http://sea.unep-wcmc.org/wdbpa/index.htm>. World Wide Web.